

HW #11 (221A), due Dec 3, 4pm

1. Work out the first-order shifts in energies of $2s$ and $2p$ states of the hydrogen atom due to the relativistic corrections, the spin-orbit interaction, and the so-called Darwin term,

$$-\frac{(\vec{p})^2}{8m_e^3c^2} + g\frac{1}{4m_e^2c^2}\frac{1}{r}\frac{dV_c}{dr}(\vec{L}\cdot\vec{S}) + \frac{\hbar^2}{8m_e^2c^2}\Delta V_c, \quad V_c = -\frac{Ze^2}{r}. \quad (1)$$

At the end of the calculation, take $g = 2$ and evaluate the energy shifts numerically.

2. Take the simple harmonic oscillator $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2$, and the perturbation $V = \frac{1}{2}\epsilon m\omega^2x^2$. Calculate the correction to the ground-state wave function up to the second order in perturbation, and compare it with the exact result.
3. Calculate the correction to the hydrogen ground-state energy due to a constant magnetic field up to $O(B^2)$. Ignore the proton spin. Use the symmetric gauge.
4. Using the polarizability of hydrogen atom discussed in Sakurai Eq. (5.1.73), calculate the index of refraction of the hydrogen gas (stp). Assume that the polarizability of the hydrogen molecule is simply twice that of the hydrogen atom. Compare it to the data of $n - 1 = 0.000140$ ($\lambda \simeq 590$ nm) and discuss the source(s) of the small discrepancy.