HW #7 (221A), due Oct 29, 4pm

1. For a free particle $H = \frac{\vec{p}^2}{2m}$, show that all $\vec{p}$ and $\vec{L}$ are conserved. What symmetries are behind these six conservation laws?

2. For a particle in a potential that depends only on z, $H = \frac{\vec{p}^2}{2m} + V(z)$, what are the remaining symmetries of the system, and the corresponding conservation laws? Show that they are indeed conserved using the Heisenberg equation of motion.

3. Write down the representation matrices for J_z, J_+, J_- for $j = 1$, $5/2$, 4, $9/2$, and verify their commutation relations $[J_z, J_\pm] = \pm\hbar J_\pm$, $[J_+, J_-] = 2\hbar J_z$.

4. Verify $L_+ Y_l^l = 0$ and $L_- Y_l^m = \sqrt{(l+m)(l-m+1)}\; Y_l^{m-1}$ for all m when $l = 2$.

5. Plot the "shape" of spherical harmonics for z, $x + iy$, $x^2 - y^2$, yz, $x^2 + y^2 - 2z^2$, $(x+iy)^3$, $x^3 - 3xy^2$, $z(x^2 - y^2)$, $(5z^2 - 3r^2)z$ orbitals as defined in the class.