HW #2 (221B), due Feb 2, 4pm

- 1. Use the solution for $\psi(x)$ from the Lippmann–Schwinger equation in the case of the one-dimensional delta function potential you studied in HW #1. Form a Gaussian wave packet of the solution with the factor $e^{-(k-q)^2d^2}$ and study its time evolution. Mathematica can do the integration over the wave numbers analytically in terms of the error function with complex arguments. Plot the probability density in space at different time by choosing appropriate parameters. Show that only the incident wave exists at $t \ll 0$, while both transmitted and scattered wave exist at $t \gg 0$. Around $t \sim 0$, observe the interference between the incident and the scattered waves.
- 2. Consider the scattering problem by the Yukawa potential

$$V = V_0 \frac{e^{-r/a}}{r} \tag{1}$$

in three dimensions.

- (a) Calculate the scattering amplitude and the total cross section using Born approximation.
- (b) Discuss the validity of Born approximation for the Yukawa potential, by requiring

$$\frac{2m}{\hbar^2} \left| \int d\vec{x} \frac{e^{ikr}}{4\pi r} V(\vec{x}) e^{ikz} \right| \ll 1.$$
(2)

(c) Show that the total cross section is smaller than the "geometric cross section" $\sim 4\pi a^2$ when Born approximation is valid independent of the momenta.