Homework #2 (221b)
Problem 1.

Set $\hbar$=1. Overall normalization is arbitrary. In the scattering amplitude f(q', q) we make the approximation q=k. It is useful to play around with different values for γ, k, d, m to see how the scattering changes; e.g. γ=10 means the incoming wave is almost totally reflected. To see the movie, double-click any of the graphs.

This solution and all of yours participate in a little quantum mechanical fraud. Wavepacket spreading goes as $\hbar t/md^2$, and for small enough d you can see it within the time frame of the movie. But for negative times the packet actually unspreads as time increases. This is wrong. Can you see where the problem enters and how to fix it?

One detail that has confused some people--the q integral should run over (-∞, ∞). We are expanding our wavefunction in a complete set of momentum eigenstates with gaussian weighting. The major contribution comes from q=k, but there are wavevectors of all q in our packet, includes negative ones. Letting the q integral run only over (0,∞) is also a valid thing to do, but it produces a different sort of wavepacket. (For a gaussian sharply peaked or translated far from the origin in q space, the difference is only in the tail of the gaussian, hence negligible.)

```
scattered = Integrate[(-I m γ)/(k + I m γ) E^(I q r) E^(-I q^2 t/(2 m)) E^(-(q-k)^2 d^2), {q, -∞, ∞}]
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$$-\frac{i\, e^{-\frac{m r^2+2 i d^2 k(-2 m r+k t)}{4 d^2 m+2 i t}}\, m\sqrt{2\pi}\,\gamma}{\sqrt{2 d^2+\frac{i t}{m}}\,(k+i m\gamma)}$$

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incoming = Integrate[E^(I q x) E^(-I q^2 t/(2 m)) E^(-(q-k)^2 d^2), {q, -∞, ∞}]
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$$\frac{e^{-\frac{m x^2+2 i d^2 k(k t-2 m x)}{4 d^2 m+2 i t}}\sqrt{2\pi}}{\sqrt{2 d^2+\frac{i t}{m}}}$$

```
ψ = incoming + scattered;

Do[Plot[Abs[ψ]^2 /. {k → 1, m → 1, d → 5, γ → 1, r → Abs[x], t → 2 i},
  {x, -90, 90}, PlotRange → {0, .1}], {i, -30, 30}]
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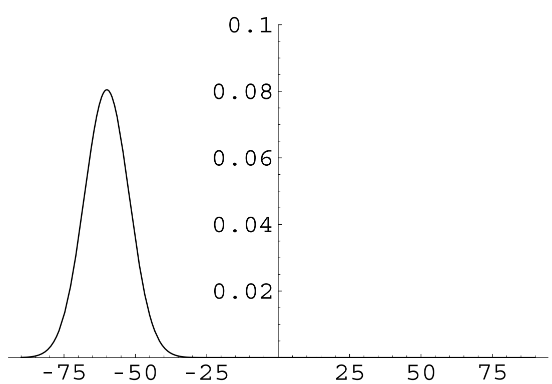

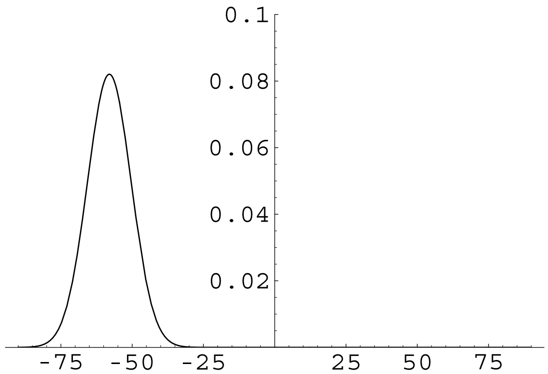
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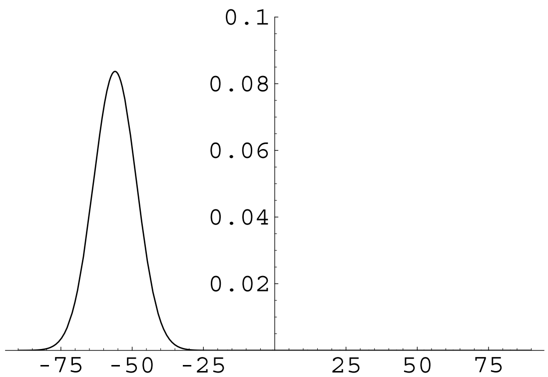
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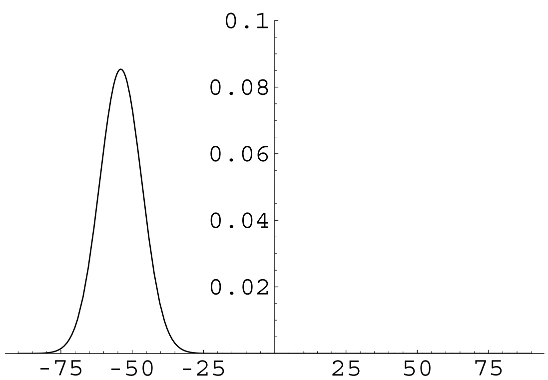
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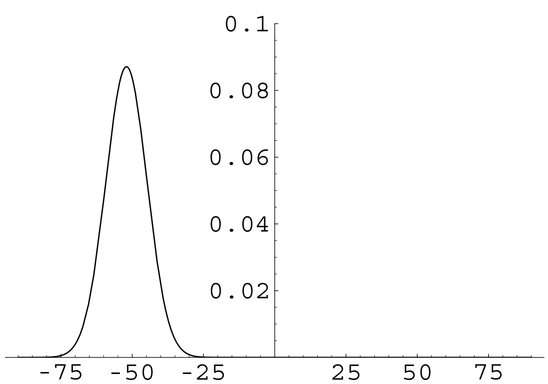
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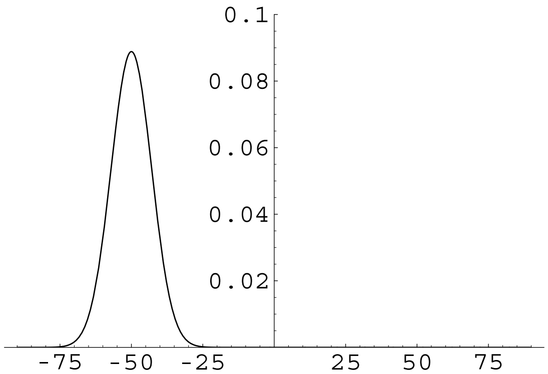
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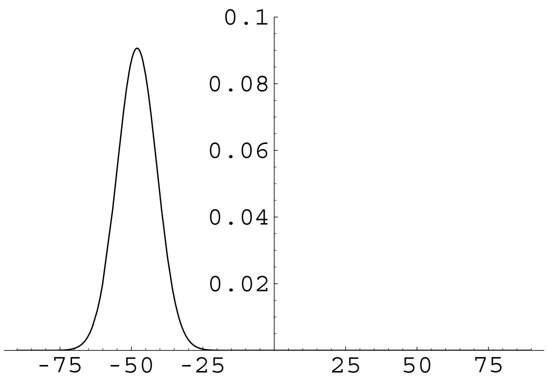
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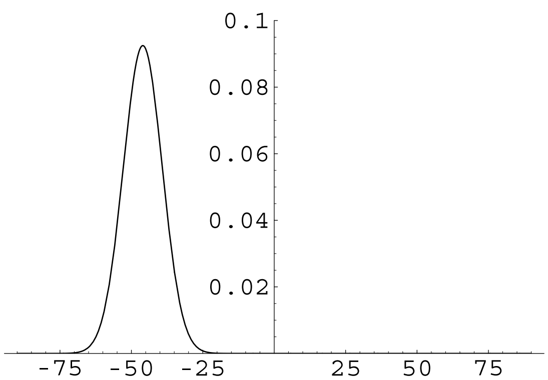
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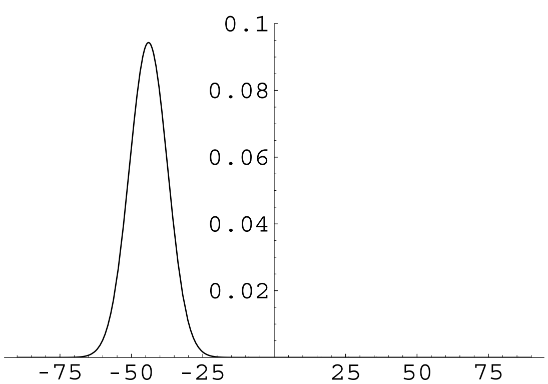
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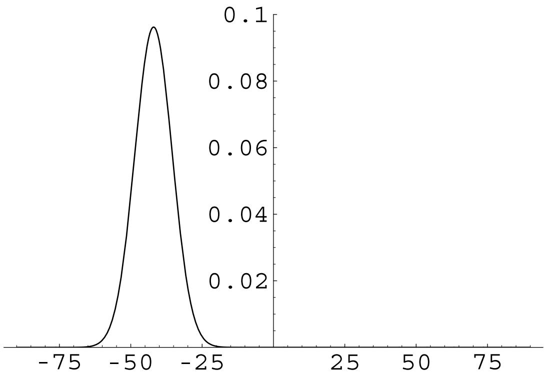
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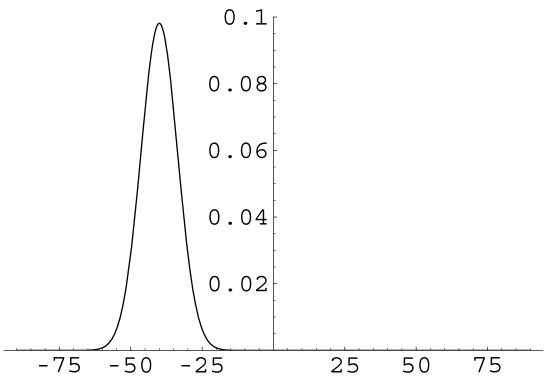
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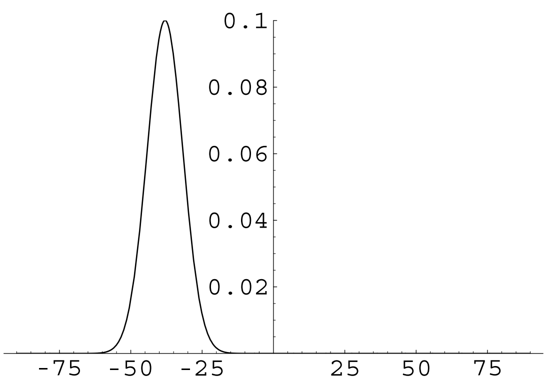
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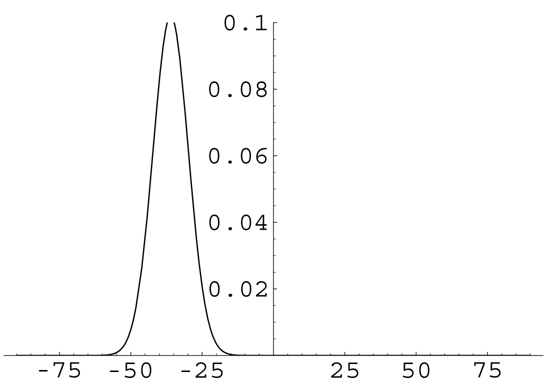
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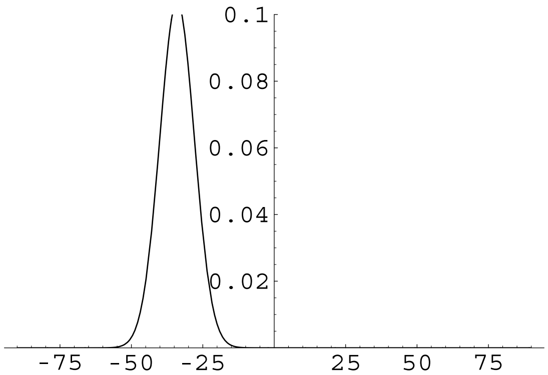
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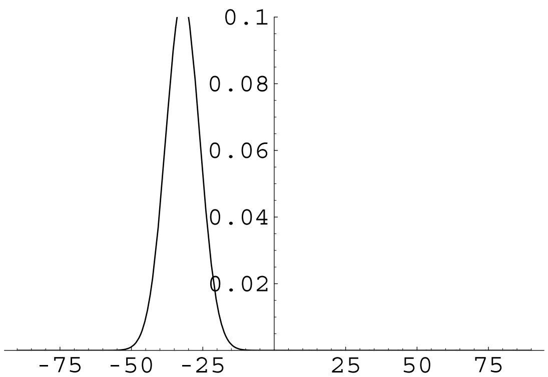
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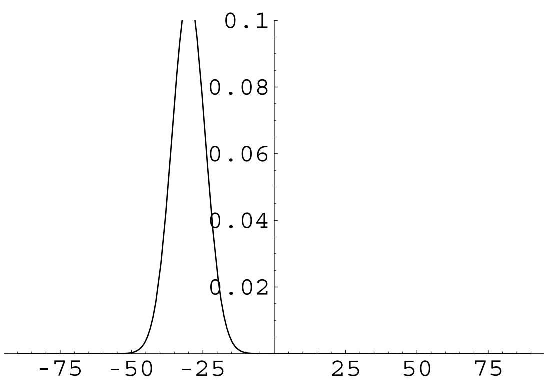
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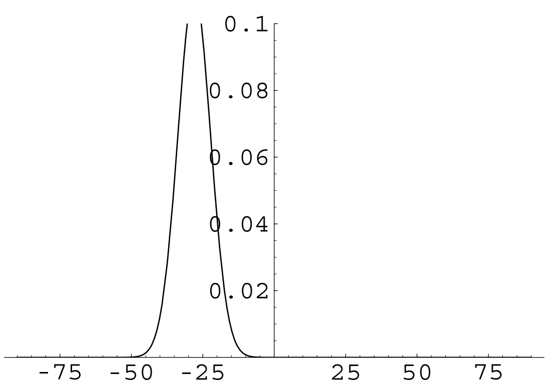
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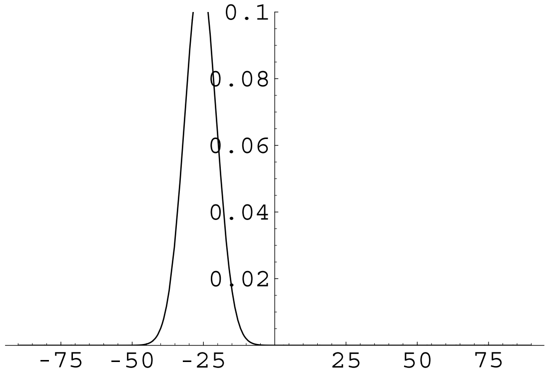
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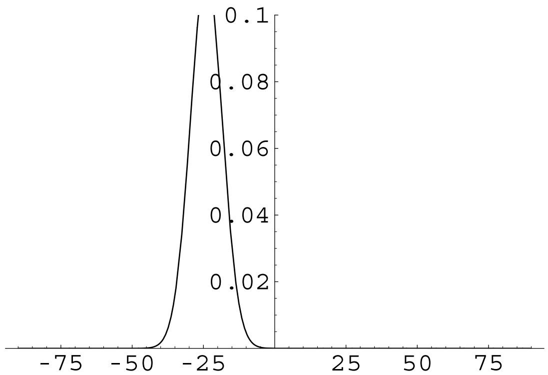
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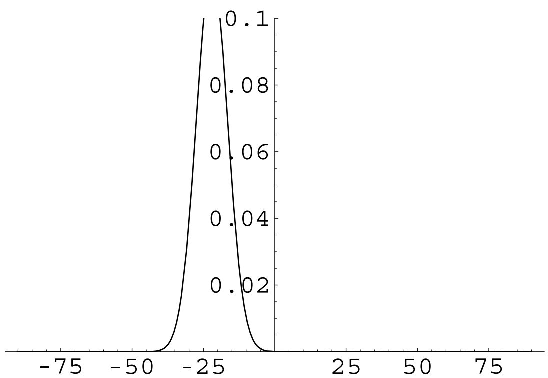
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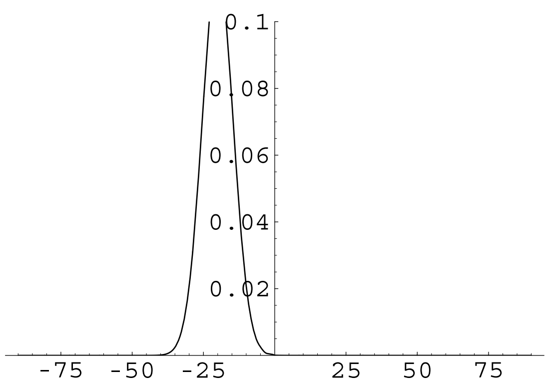
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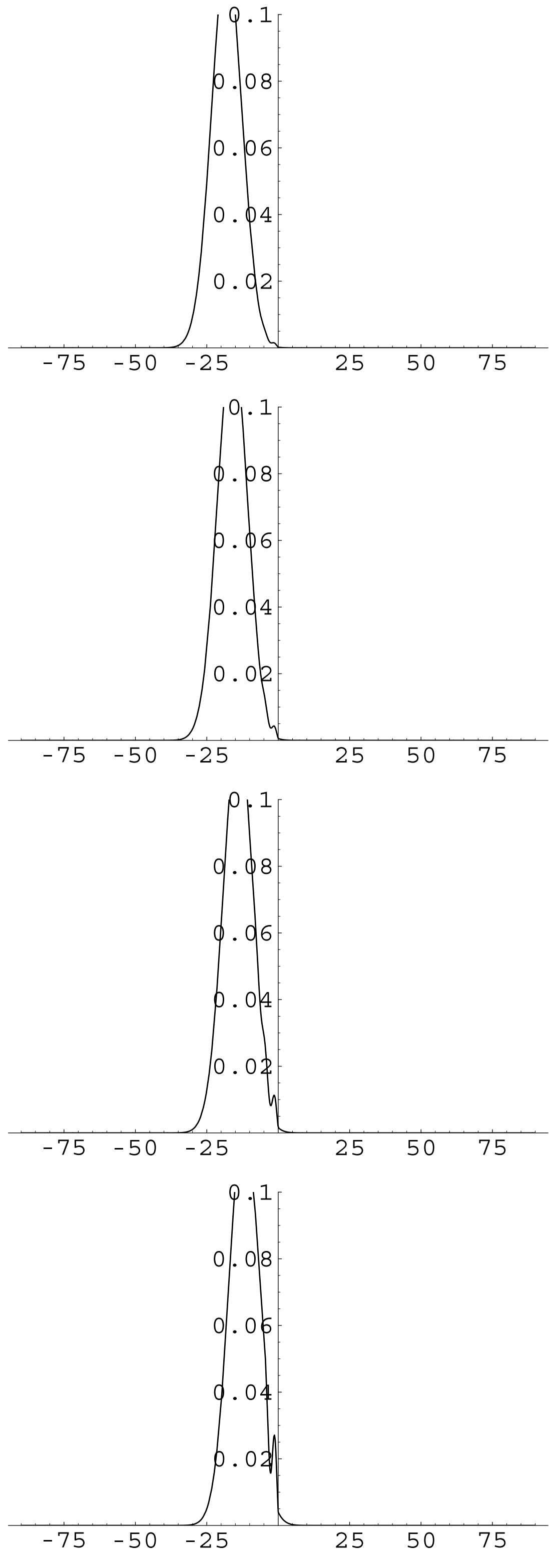
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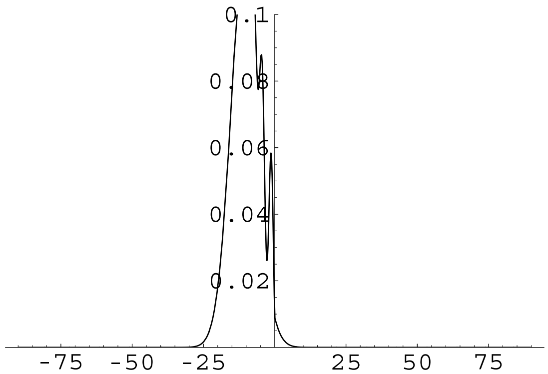
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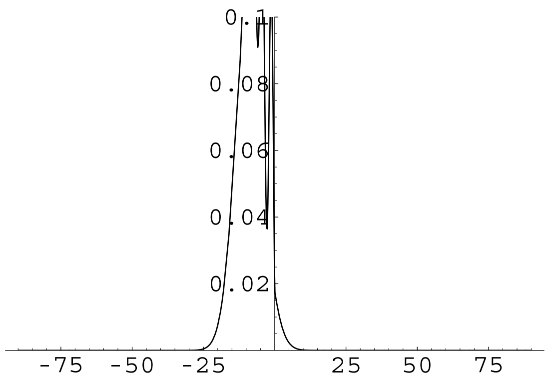
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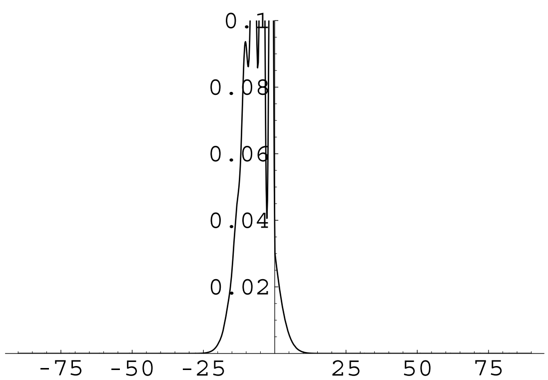
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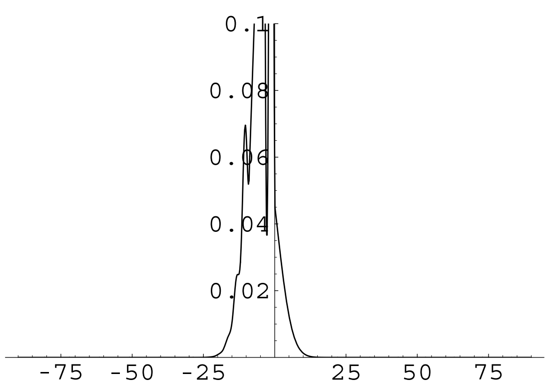
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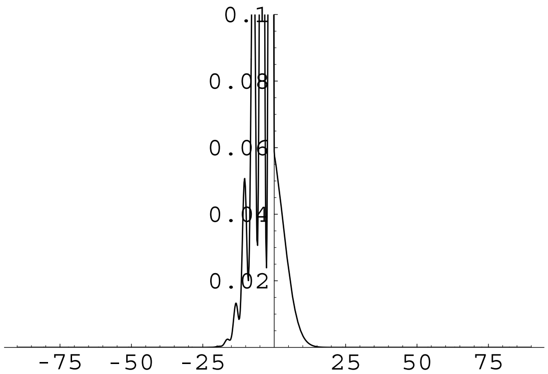
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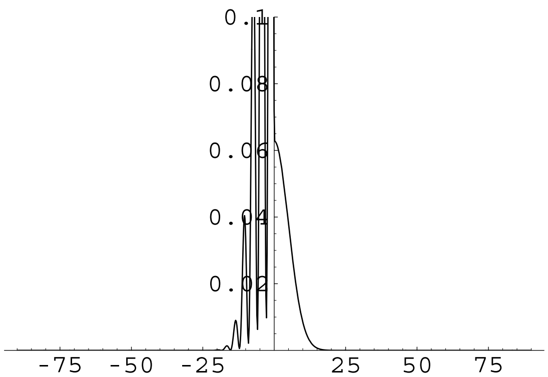
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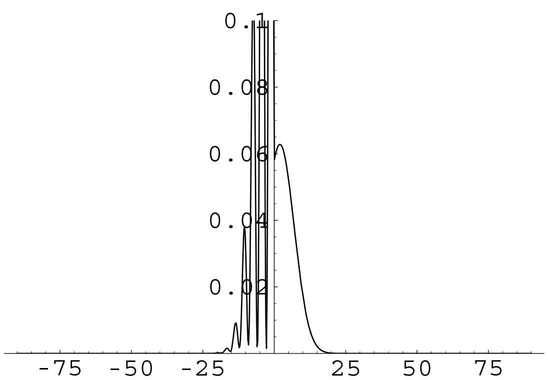
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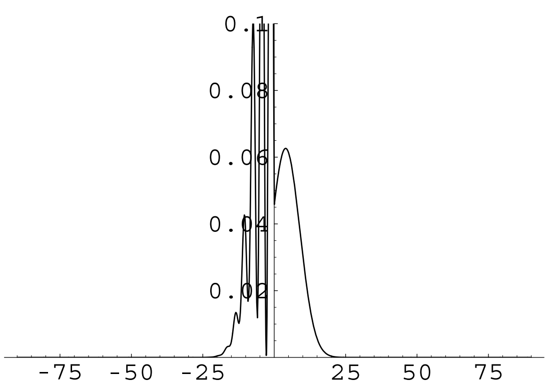
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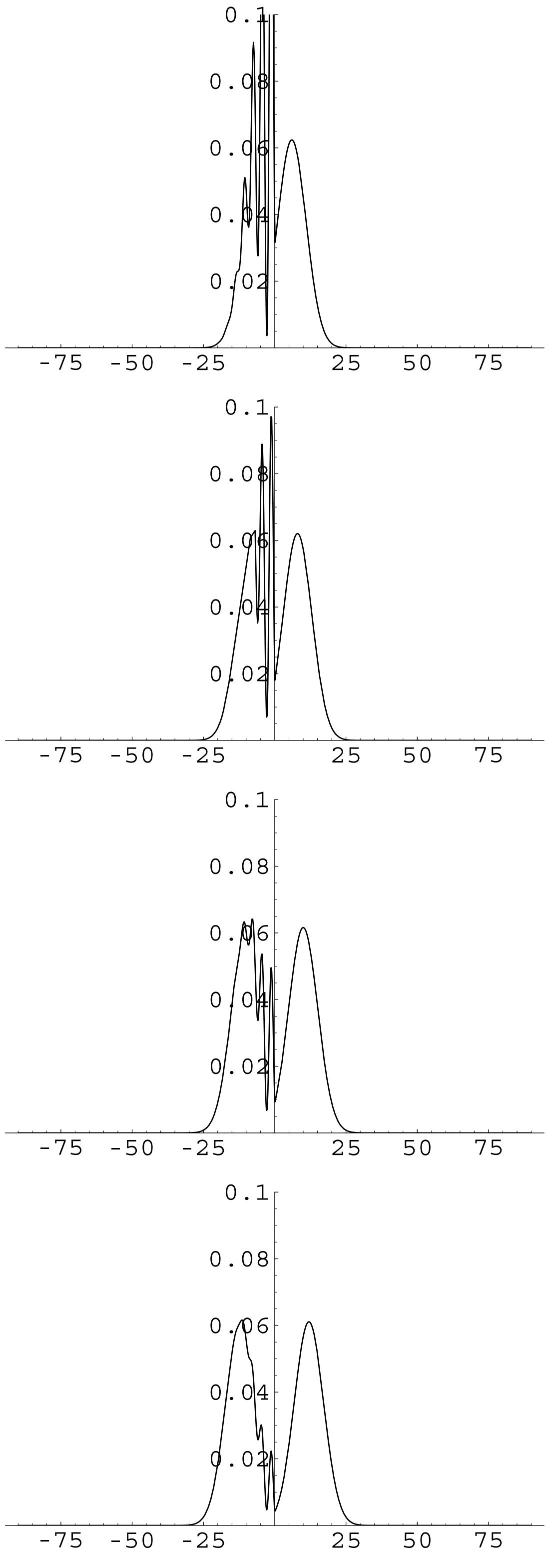
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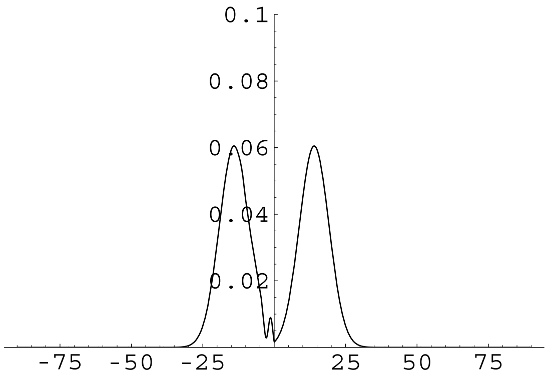
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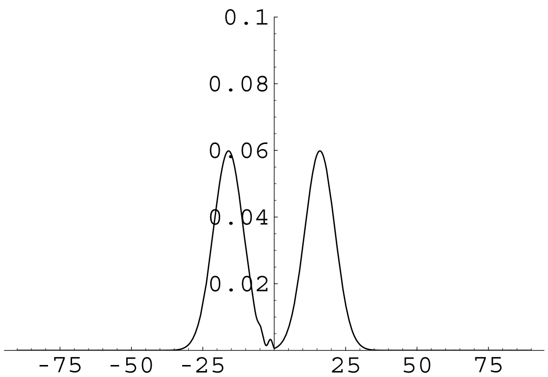
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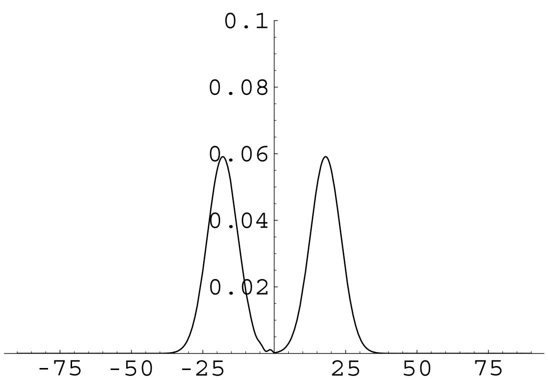
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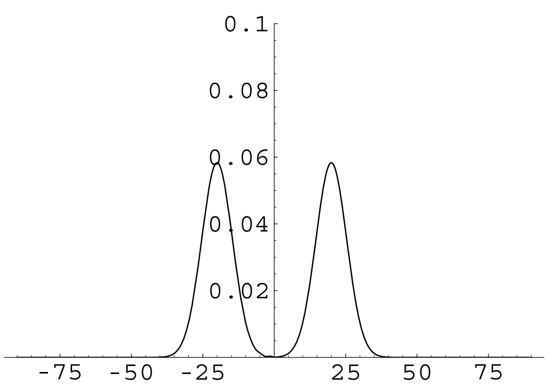
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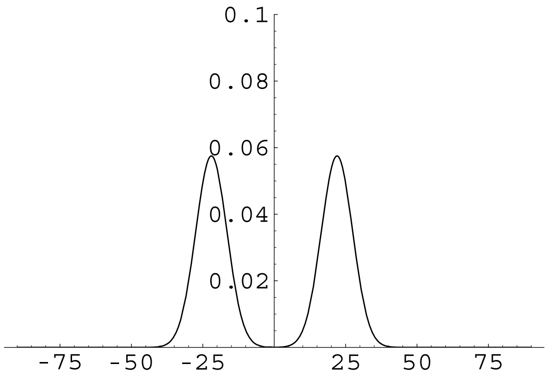
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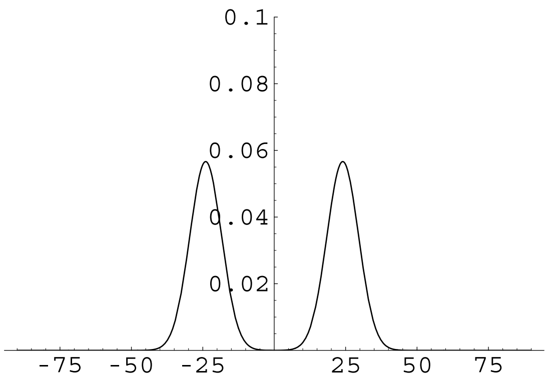
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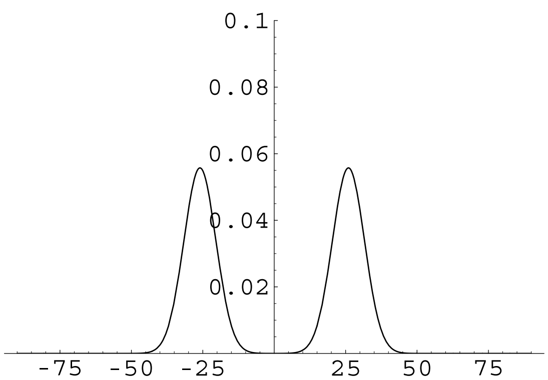
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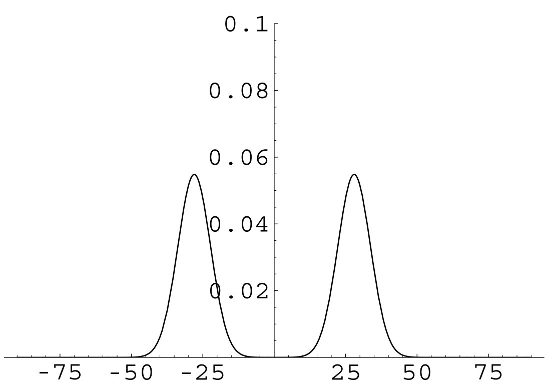
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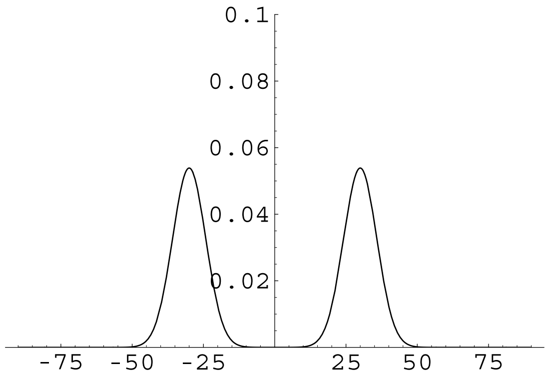
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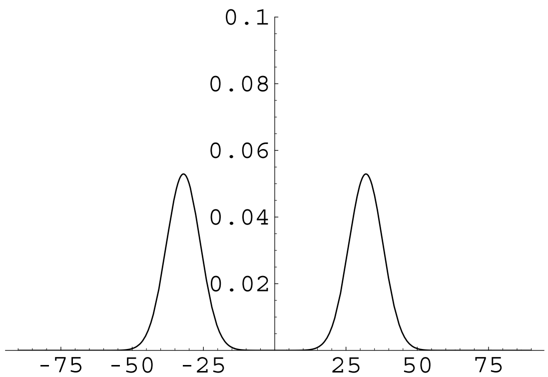
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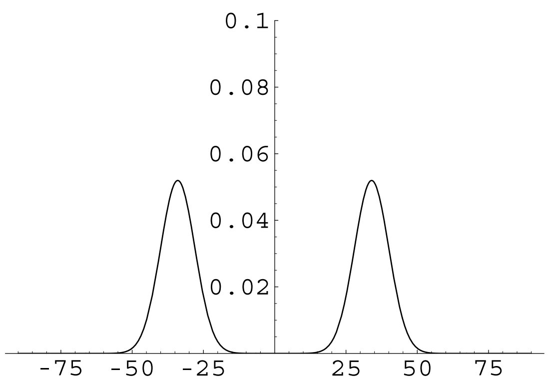
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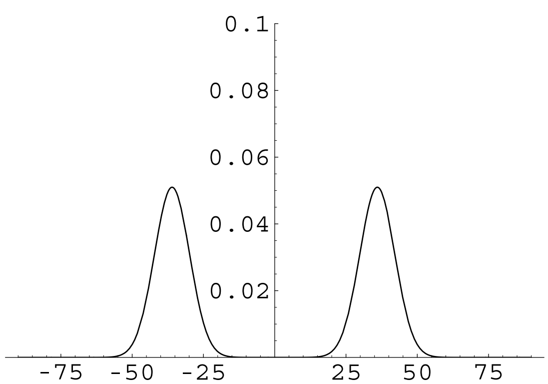
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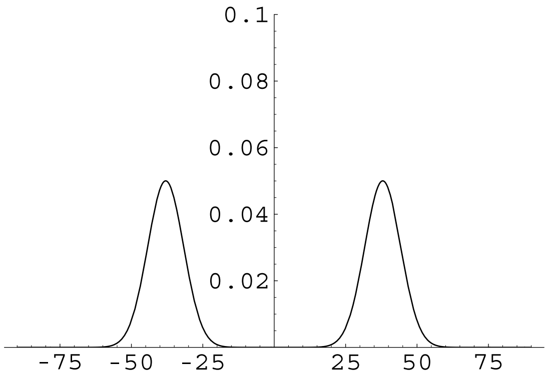
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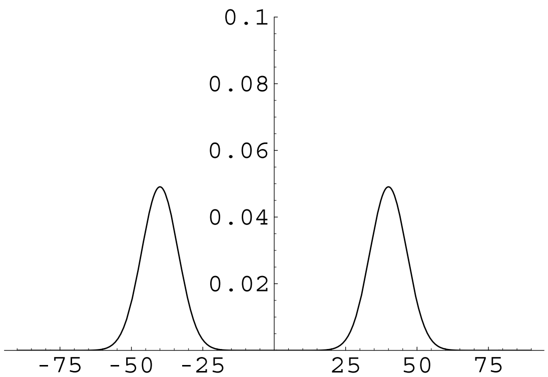
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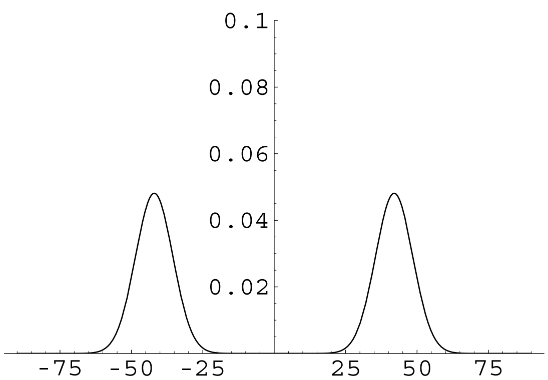
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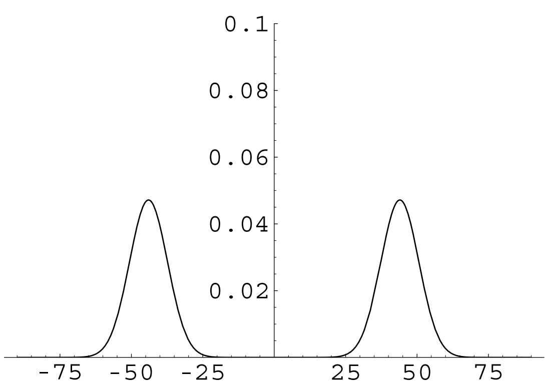
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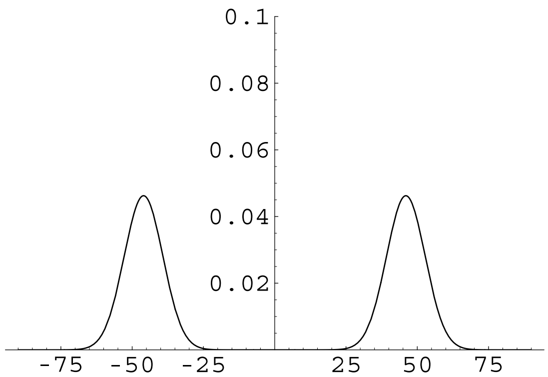
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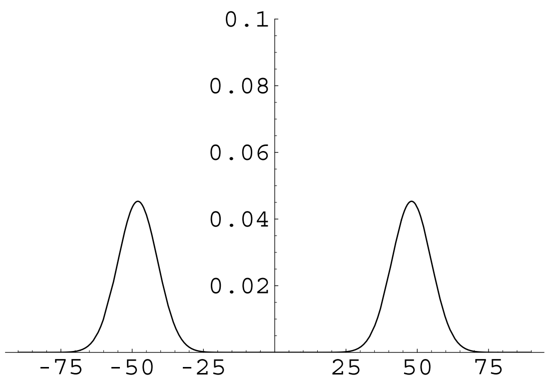
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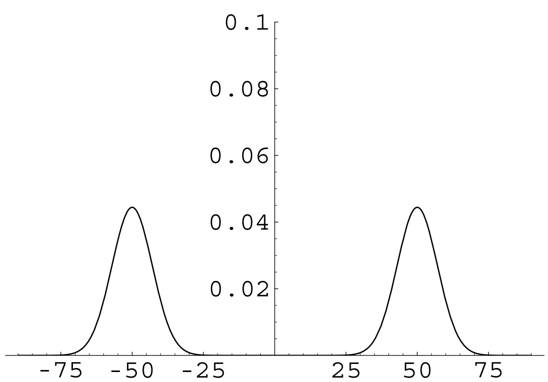
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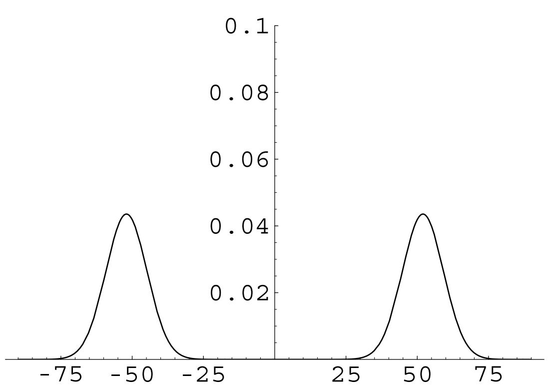
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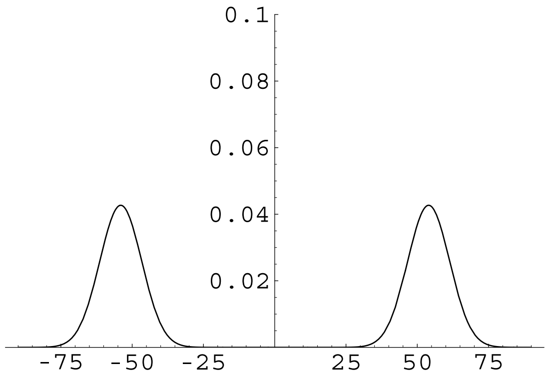
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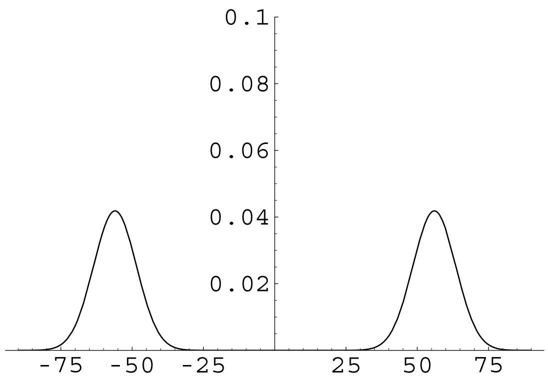
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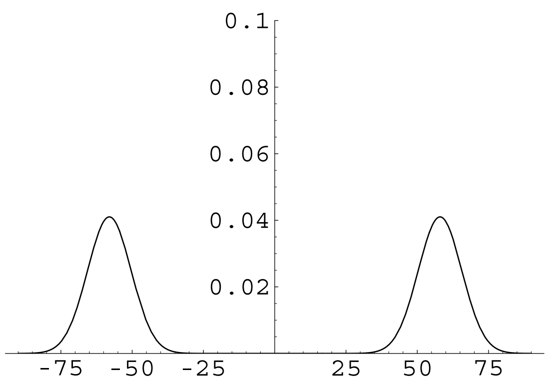
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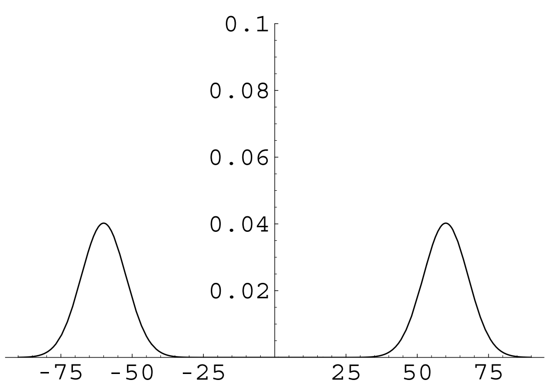
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