# HW #1 (221B), due Feb 1, 5pm

1. Show that Lippmann–Schwinger equation in one dimension is given by

$$\psi(x) = \frac{e^{ikx}}{\sqrt{2\pi\hbar}} + \frac{-mi}{\hbar^2 k} \int dx' e^{ik|x-x'|} V(x') \psi(x'). \tag{1}$$

   $k > 0$ is assumed. The first term corresponds to the incident particle, while the second term the scattered wave.

2. For large $r = |x| \gg a$, and assuming that the scattering potential $V(x')$ is sizable only in a small region $|x'| \lesssim a$, write down the asymptotic form of the wave function. Use $|x - x'| = \sqrt{(x-x')^2} = r - x'x/r + O(x')^2$, and rewrite the wave function in the form

$$\begin{aligned} \psi(x) &= \frac{1}{\sqrt{2\pi\hbar}} \left[ e^{ikx} + f(k,k') e^{ikr} \right] \\ &= \frac{1}{\sqrt{2\pi\hbar}} \left\{ \begin{array}{ll} (1 + f(k,k)) e^{ikx} & x > 0 \\ e^{ikx} + f(k,-k) e^{-ikx} & x < 0 \end{array} \right. , \end{aligned} \tag{2}$$

   and work out what $f(k, k')$ is. Compared to the three-dimensional case, the function $f(k, k')$ takes only two values, $k' = \pm k$, because of only one dimension.

3. Work out the probability current density for the plane wave and the spherical wave

   (a) $\psi(\vec{x}) = e^{i\vec{k}\cdot\vec{x}}$

   (b) $\psi(\vec{x}) = \frac{e^{ikr}}{r}$

   Show the flow of probability graphically (*e.g.*, `PlotVectorField` in Mathematica). Also verify that the probability current is divergenceless except at the origin for the spherical wave.