Physics 221B: Solution to HW # 7
Nuclear Physics[1]

1) ^{210}Po and the Nuclear Shell Model

$^{210}_{84}$Po has 126 neutrons and 84 protons. 126 is a magic number, which means the neutrons form a complete shell. Exciting a neutron will cost a lot of energy so the low lying excitations will be due to excited protons. 82 is also a magic number, so the low lying excitations will be due to the 'extra' two protons.

The two 'valance' protons are in the state $1h_{9/2}$ which is part of the $5\hbar\omega$ oscillator state, with odd parity. The lowest lying states will have both protons in $1h_{9/2}$. Adding angular momentum in the usual way ,these states will have a total angular momentum of

$$J = \frac{9}{2} \times \frac{9}{2} = 0, 1, 2, \ldots, 9 \tag{1}$$

and will have odd×odd=even parity. Next, we demand the fermionic wavefunction for the two protons be anti-symmetric. Since both protons are in the same energy level and with the same J (they differ only in J_z) anti-symmetry will disallow certain values of total angular momentum. Specifically we demand that if we interchange $J_{1,z}$ and $J_{2,z}$, the wavefunction will change sign. Recall that when adding angular momentum of two identical states with the Clebsh-Gordon procedure the highest J state is always symmetric (for all J_z), the second highest will be anti-symmetric, and so on. In our case, the possible values of J from Eq. (1) reduce to

$$J = 0^+, 2^+, 4^+, 6^+, 8^+. \tag{2}$$

Indeed, these are the five lowest states according to the data given. We can claim to explain the order by the tendency of protons to pair with their time-reversed state. The $J = 0$ state, with the angular momentum of the protons anti-aligned, is precisely such a time-reversed pair and is therefore the ground state. The 8^+ state is the least favored by this effect, with $2^+, 4^+$ and 6^+ in between.

The next excited levels will be due to exciting one of the protons to the next level which is $2f_{7/2}$ with odd parity as well. The parity of the next

[1] This problem set was graded out in the sun, somewhere in the Berkeley hills. I highly recommend such grading picnics for those of you who grade often. This can also explain any dead insects you might have found in your problem set.

excited states will still be odd×odd=even. Adding angular momentum, we expect the next states to have

$$J = \frac{9}{2} \times \frac{7}{2} = 1, 2, \ldots, 8. \tag{3}$$

Since the two protons are in two different energy levels we have the extra ambiguity of which proton is in which level. Anti-symmetry can be achieved by writing a Staler determinant every time we write a product of two states. Therefore all of the states above are allowed by anti-symmetry considerations. (including 1^+, contrary to what many of you said, trying to explain its absence in the spectrum). Indeed the states 2^+ through 8^+ appear next in the spectrum but in a peculiar order (disregarding states with '±' parity). I don't know of a good argument for the ordering of these states but apparently even J is favored. The state 1^+ is absent, with no obvious reason. A possible guess is that it is disfavored by the effects that determine the ordering of these states and that it is pushed higher up in the spectrum, outside of the range we were given.

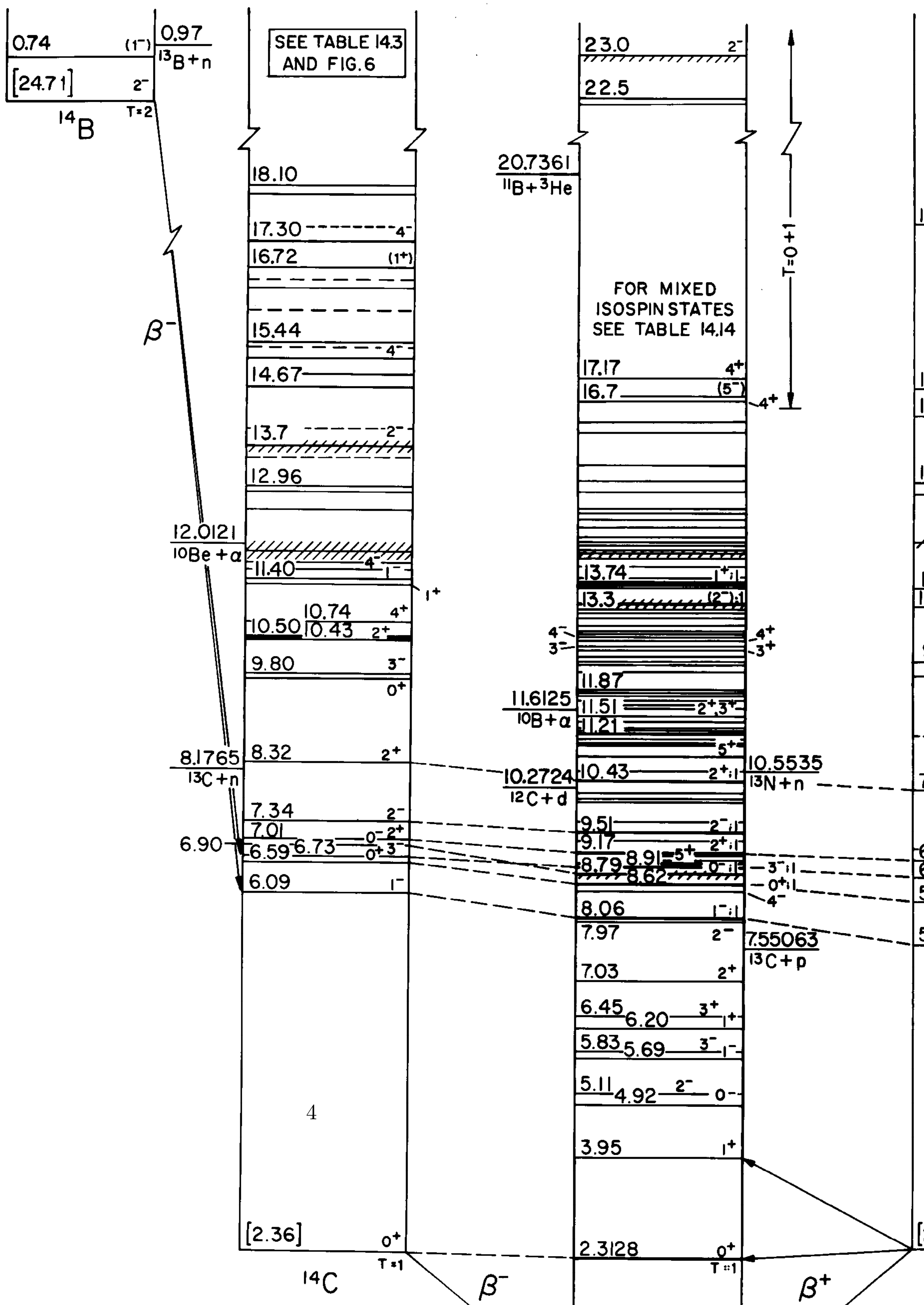
2) Iso-Multiplets of ^{14}C, ^{14}N and ^{14}O

All three nuclei discussed have 14 nucleons and therefore are connected through 'isospin rotations'. It makes sense to classify the different excitations of these nuclei into multiplets with a certain isospin. Carbon and Oxygen have an isospin in the z direction of $I_z = +1$ and $I_z = -1$ respectively. Clearly they cannot be part of an $I = 0$ multiplet (singleplet). Nitrogen, on the other hand, has $I_z = 0$ and therefore can be part both $I = 0$ or $I = 1$ multiplets.

How can we figure out which states go with which? This question of course applies only to the $I = 1$ case because as we already stated, the $I = 0$ singleplets will only be states of Nitrogen *i.e.* excitations of ^{14}N that are not part of an $I = 1$ multiplet will have $I = 0$.

The most simple criterion for finding candidates for isospin multiplets is demanding they have similar non-iso quantum numbers (J and parity). Since isospin generators commute with everything else, we can't expect to change angular momentum or parity with an iso-rotation. The second but more loose criterion is to have matching energy levels. This is obviously not exact, since isospin is not an exact symmetry, but we can generally expect the multiplets to appear in the same order in energy. Note that in the data given the energy for each nucleus is counted from the ground state of that nucleus. If we want the last criterion to roughly work we need to shift the energies correspondingly *e.g.* set the first $I = 1$ multiplet degenerate, or lookup what the ground state energies really are.

The following plot was taken form the TUNL Nuclear Data Project website at `http://www.tunl.duke.edu`. I thank Jeff Vieregg for referring me to this website in his solution.



0.74 (1⁻)
[24.71] 2⁻
T=2
14B
0.97
13B+n
β⁻
SEE TABLE 14.3 AND FIG. 6
18.10
17.30 4⁻
16.72 (1⁺)
15.44 4⁻
14.67
13.7 2⁻
12.96
12.0121
10Be+α
11.40 4⁻ 1⁻
1⁺
10.74 4⁺
10.50 10.43 2⁺
9.80 3⁻
0⁺
8.1765
13C+n
8.32 2⁺
7.34 2⁻
7.01 2⁺
6.90
6.59 6.73 0⁻ 3⁻ 0⁺
6.09 1⁻
[2.36] 0⁺
T=1
14C
β⁻
23.0 2⁻
22.5
20.7361
11B+3He
FOR MIXED ISOSPIN STATES SEE TABLE 14.14
T=0+1
17.17 4⁺
16.7 (5⁻)
4⁺
13.74 1⁺;1
13.3 (2⁻);1
4⁻ 3⁻
4⁺ 3⁺
11.87
11.6125
10B+α
11.51 2⁺,3⁺
11.21
5⁺
10.2724
12C+d
10.43 2⁺;1
10.5535
13N+n
9.51 2⁻;1
9.17 2⁺;1
8.91 5⁺
8.79 0⁻;1
8.62
3⁻;1
0⁺;1
4⁻
8.06 1⁻;1
7.97 2⁻
7.55063
13C+p
7.03 2⁺
6.45 6.20 3⁺ 1⁺
5.83 5.69 3⁻ 1⁻
5.11 4.92 2⁻ 0⁻
3.95 1⁺
2.3128 0⁺
T=1
β⁺