129A HW # 5 (due Oct 10)

Pions (π^+, π^0, π^-) form an $I = 1$ multiplet, while nucleons (p, n) $I = 1/2$. A $|\pi N\rangle$ state contains both $I = 3/2$ and $I = 1/2$ states. Answer the following questions.

1. Write down the states $|3/2, 3/2\rangle$, $|3/2, 1/2\rangle$, $|3/2, -1/2\rangle$, $|3/2, -3/2\rangle$, $|1/2, 1/2\rangle$, $|1/2, -1/2\rangle$ in terms of pions and nucleons. (Hint: this is exactly the same as the addition of two angular momenta 3/2 and 1/2 in quantum mechanics. A keyword to look for is Clebsch–Gordan coefficients.)

2. Write down $|\pi^+ p\rangle$, $|\pi^- p\rangle$, $|\pi^0, n\rangle$ states in terms of isospin eigenstates.

3. A resonance can be approximated by the Breit–Wigner amplitude

$$\mathcal{A}_{BW} = \frac{-\Gamma_r/2}{E_i - E_r + i\Gamma_r/2}, \tag{1}$$

where E_i is the initial energy, E_r is the resonance energy and Γ_r is a parameter with dimension of energy which characterizes how wide the resonance is. Depict the function $|\mathcal{A}_{BW}|^2$ to see that it shows a peak at $E_i = E_r$.

4. An interpretation of a resonance is creation of a short-lived quantum mechanical state. To obtain the time-dependence of such a state, perform the Fourier transform

$$\int_{-\infty}^{+\infty} \frac{dE_i}{2\pi\hbar} \mathcal{A}_{BW} e^{-iE_i t/\hbar} \tag{2}$$

and show that it has expontentially decaying behavior with life time $\tau = 1/\Gamma_r$.

5. When pions and nucleons scatter through Δ resonances, the scattering amplitudes are proportional to

$$\langle \pi N|\Delta\rangle \frac{1}{E_i - m_\Delta c^2 + i\Gamma_\Delta/2} \langle \Delta|\pi N\rangle. \tag{3}$$

Work out the ratios of the amplitudes for the following three processes:

$$\pi^+ p \to \Delta^{++} \to \pi^+ p \tag{4}$$
$$\pi^- p \to \Delta^0 \to \pi^- p \tag{5}$$
$$\pi^- p \to \Delta^0 \to \pi^0 n \tag{6}$$

Note that the Δ resonances have definite isospin $I = 3/2$.

6. Write down the ratios of cross sections at the Δ resonances.