HW # 8 Solutions

1. Lifetime of K_S is $\tau_S = (0.8927 \pm 0.0009) \times 10^{-9}$ sec, and the width is given by its inverse, $\Gamma_S = \hbar/\tau_S = 7.37 \times 10^{-15}$ GeV. The branching fraction of $K_S \to \pi^+\pi^-$ is $\mathrm{BR}(K_S \to \pi^+\pi^-) = (68.61 \pm 0.28)\%$, and hence the partial width is given by $\Gamma(K_S \to \pi^+\pi^-) = \Gamma_S \mathrm{BR}(K_S \to \pi^+\pi^-) = 5.06 \times 10^{-15}$ GeV.

2. From the formula for the partial widths, the quantity in question

$$\int d\Phi_{\pi\pi} |\langle \pi^+\pi^- | \mathcal{H}_{weak} | K_1 \rangle|^2, \tag{1}$$

is the same as $2m_{K_S}\Gamma(K_S \to \pi^+\pi^-)$. Therefore, it is given by $2 \times 0.497672 \times 5.06 \times 10^{-15} = 5.04 \times 10^{-15}$ GeV^2.

3. $\Gamma(K_L \to \pi^+\pi^-) = (\hbar/\tau_L)\mathrm{BR}(K_L \to \pi^+\pi^-) = 2.63 \times 10^{-20}$ GeV.

4. The quantity in question is the same as $2m_{K_L}\Gamma(K_L \to \pi^+\pi^-) = 2.62 \times 10^{-20}$ GeV^2.

5. Since the violation of CP lies in the small mixture of K_1 state inside K_L, the matrix element is given by

$$\langle \pi^+\pi^- | \mathcal{H}_{weak} | K_L \rangle = \epsilon \langle \pi^+\pi^- | \mathcal{H}_{weak} | K_1 \rangle. \tag{2}$$

Therefore,

$$\int d\Phi_{\pi\pi} |\langle \pi^+\pi^- | \mathcal{H}_{weak} | K_L \rangle|^2 = \int d\Phi_{\pi\pi} |\langle \pi^+\pi^- | \mathcal{H}_{weak} | K_1 \rangle|^2 \epsilon^2, \tag{3}$$

and using the calculated numbers above, we find

$$\epsilon = \sqrt{2.62 \times 10^{-20}\ \mathrm{GeV}^2 / 5.04 \times 10^{-15}\ \mathrm{GeV}^2} = 2.28 \times 10^{-3}. \tag{4}$$

6. We use the branching fraction $\mathrm{BR}(K_S \to \pi^0\pi^0) = (31.39 \pm 0.28)\%$. Then following the similar analysis as above, we obtain

$$\mathrm{BR}(K_L \to \pi^0\pi^0) = \epsilon^2 \Gamma_S \mathrm{BR}(K_S \to \pi^0\pi^0)/\Gamma_L = 9.45 \times 10^{-4}. \tag{5}$$

Here we used the fact that m_{K_L} and m_{K_S} are the same within the accuracy needed for this calculation. This is completely consistent with the measured value $(9.36 \pm 0.20) \times 10^{-4}$.