HW # 9 Solutions

1. The formula for the total cross section of $e^+e^- \to \mu^+\mu^-$ we calculated is $\sigma = \frac{4\pi\alpha^2}{3s}$. Since s has a dimension of energy squared, we use $(\hbar c)^2$ with dimension (energy·length)^2 to convert it to the cross section. Together with $\alpha = 1/137.0$ from page 2 in the booklet, we find

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{86.8 \text{ nb}}{(\sqrt{s}/\text{GeV})^2}. \tag{1}$$

(This is called "point cross section" and is a formula which every particle physicist should remember!) With a luminosity of 10^{31} cm^{-2} sec^{-1}, the number of events per hour is

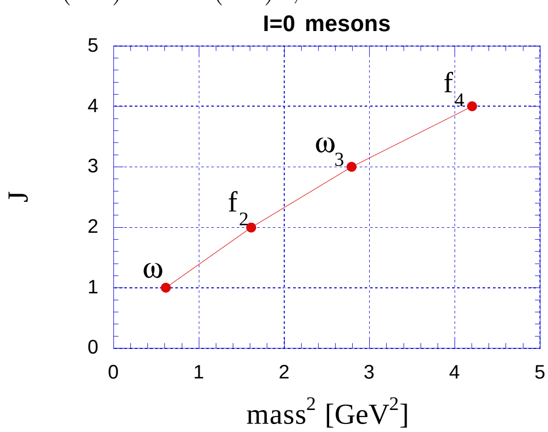
$$\frac{86.8 \times 10^{-9} \times 10^{-24} \text{ cm}^2}{60^2} \times 10^{31} \text{ cm}^{-2} \text{ sec}^{-1} \times 3600\text{sec} = 0.868. \tag{2}$$

It doesn't happen even once an hour! The similar rate for the $u\bar{u}$ pair is given by the above number multiplied by the charge squared and number of colors, $0.868 \times (2/3)^2 \times 3 = 1.157$, and for the $d\bar{d}$ pair $0.868 \times (-1/3)^2 \times 3 = 0.289$. The total cross section of producing hadrons is given by the sum of $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, $c\bar{c}$ and $b\bar{b}$,

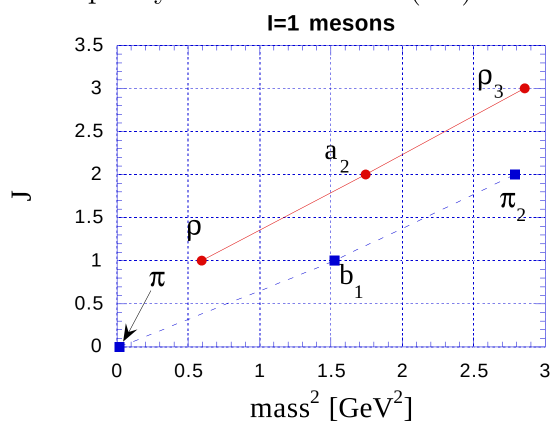
$$0.868 \times \left(\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 \right) \times 3 = 3.18. \tag{3}$$

2. The confining linear potential.

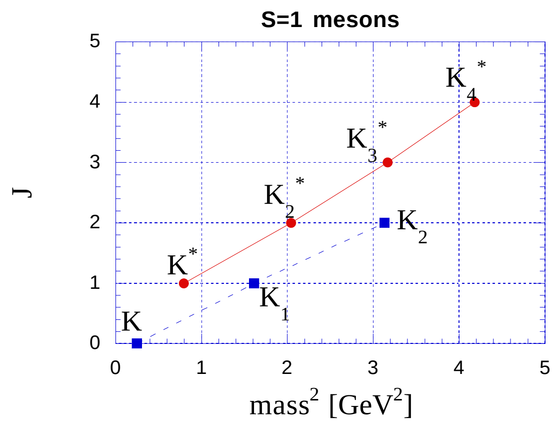
(a) These are $q\bar{q}$ bound states with total spin $S = 1$ and relative orbital angular momentum L and hence $J = L + 1$. The parity is therefore given by $P = (-1)^{L+1} = (-1)^J$, consistent with J^P quoted.



(b) The first three are $q\bar{q}$ bound states with total spin $S = 1$ and relative orbital angular momentum L, and $J = L + 1$ combined. The parity is therefore given by $P = (-1)^{L+1} = (-1)^J$, consistent with J^P quoted. The second three have total spin $S = 0$ and L, and hence $J = L$ combined. The parity is therefore $P = (-1)^{L+1} = (-1)^{J+1}$.



(c) The quantum numbers are determined the same way as the previous problem.



(d) The angular momentum $J = rp$ is a conserved quantum number, and we solve for p, $p = J/r$. Substituting it to the Hamiltonian, we find

$$H = \frac{J}{r} + \frac{r}{\alpha'}. \tag{4}$$

We then minimize it with respect to r,

$$\frac{dH}{dr} = -\frac{J}{r^2} + \frac{1}{\alpha'} = 0, \tag{5}$$

or, $r = (\alpha' J)^{1/2}$. Plugging this back in to the Hamiltonian, we find $H = (J/\alpha')^{1/2}$, or, $m^2 = H^2 = J/\alpha'$.