# 129B HW # 2 (due Feb 6)

The $W$-boson was first discovered at the CERN $p\bar{p}$ collider $Sp\bar{p}S$, by the UA1 experiment led by Carlo Rubbia. Now it is studied intensively both at CERN $e^+e^-$ collider LEP-II and Fermilab $p\bar{p}$ collider Tevatron.

**1.** Given the mass of the $W$-boson from the PDG booket, calculate the coupling constant $g$ of the $W$-boson, using the relation $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$.

**2.** Compare $g^2/4\pi$ with the QED fine-structure constant $\alpha = e^2/4\pi$. Which one is larger?

**3.** The partial decay rate of the $W$-boson into electron and neutrino is given by

$$\Gamma(W^- \to e^-\bar{\nu}_e) = \frac{g^2}{48\pi} m_W. \tag{1}$$

Calculate the numerical value of the predicted decay rate. Also obtain the partial decay rate from the data, and compare them.

**optional**

**a.** The amplitude of $W^- \to e^-\bar{\nu}_e$ is given by

$$i\mathcal{M} = \frac{igm_W}{2}(1 - \cos\theta)e^{i\phi}, \tag{2}$$

when $W$-boson is in the spin state $s_z = +1$ and electron momentum is $p_e^\mu = \frac{m_W}{2}(1, \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. Argue that the $\cos\theta$ dependence makes sense from the angular momentum conservation, noting that only $(e^-)_L(\bar{\nu}_e)_R$ combination is allowed. Derive Eq. (1) using the golden rule.

**b.** Work out the amplitude of the decay $W^- \to e^-\bar{\nu}_e$. Using the Feynman rule discussed in the class, the amplitude is given as

$$i\mathcal{M} = \frac{ig}{\sqrt{2}}\,\bar{u}(p_e)\gamma_\mu\frac{1-\gamma_5}{2}v(p_\nu)\,\epsilon^\mu(p_W). \tag{3}$$

Here, the "polarization vector" $\epsilon^\mu(p_W)$ is the wave function of a spin 1 boson. When the $W$-boson is at rest, it is given by either one of the followings:

$$\epsilon_+^\mu = \frac{1}{\sqrt{2}}(0, 1, i, 0), \quad \epsilon_-^\mu = \frac{1}{\sqrt{2}}(0, -1, i, 0), \quad \epsilon_0^\mu = (0, 0, 0, 1), \tag{4}$$

for $s_z = 1, -1, 0$ state, respectively. Obtain the amplitude Eq. (2) for the $s_z = 1$ $W$-boson. You can also check that the amplitudes vanish for helicity combinations except for left-handed electron and right-handed anti-electron neutrino.