129B HW # 3 (due Feb 13)

1. Check that the Dirac equation with electromagnetic vector potential A_μ,

$$[i\gamma^\mu(\partial_\mu - ieQA_\mu) - m]\,\psi = 0, \tag{1}$$

is invariant under the gauge transformation,

$$\psi \;\rightarrow\; \psi' = e^{ieQ\chi}\psi, \tag{2}$$

$$A_\mu \;\rightarrow\; A'_\mu = A_\mu + \partial_\mu\chi. \tag{3}$$

2. Depict the Langdau–Ginzburg potential for the magnet $\vec{M}$:

$$V = (T - T_c)(\vec{M}\cdot\vec{M}) + \lambda(\vec{M}\cdot\vec{M})^2 \tag{4}$$

for $T > T_c$ and $T < T_c$ separately. Minimize the potential and find that there is a *spontaneous magnetization* for $T < T_c$. (Hint: you cannot depict a potential which depends on three quantities, M^1, M^2, M^3. Drop M^3 for the moment, and try to draw the potential on the (M_1, M_2) plane.)

optional

a. Write down the Dirac equation for left-handed electron and neutrino in the presence of W-boson vector potentials in terms of the linear combinations

$$W^+_\mu = \frac{1}{\sqrt{2}}(W^1_\mu - iW^2_\mu), \tag{5}$$

$$W^-_\mu = \frac{1}{\sqrt{2}}(W^1_\mu + iW^2_\mu), \tag{6}$$

and W^3_μ. Show that the $W^\pm_\mu$ vector potentials convert electron and neutrino with each other. Argue next that W^3_μ cannot be identified with the photon.